ENEE 610
Th 10/08/20 (server down) -- Tu 10/13/20

Note Title 9/19/2020

Lossless 1-ports

Lossless Canonical forms
Higher order via semi-state
Hysteresis via op-amp; convolution

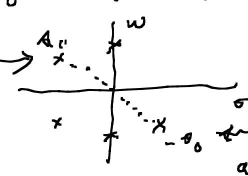
LBR = Lossless Bounded Real scattering S(s) nxn matrices
1) Rational with real coefficients
2) No poles in closed right half plane, $\sigma \geq 0$
3) $S(-s)^T S(s) = 1_n$

LPR = Lossless Positive Real admittnce Y(s) nxn matrix (also for Z or hybrid matrices)
1) Rational with real coefficients
2) No poles in open right half plane, $\sigma > 0$
3) $Y(-s)^T + Y(s) = 0_n$ almost all s {if n=1 => Ev(Y(s))=0}

An outstanding reference book for an accurate treatment of Fourier transforms and other useful advanced mathematics: (based on theory of "distributions"
Laurent Schwartz, "Mathematics for the Physical Sciences," Dover Publications, New York, 2008.

$y(s) = -y(-s)$ if LPR (driving points)

pole of $y(s)$ → s_i



is a poles of $y(-s)$ also a pole of $y(s)$ } cant be if PR } no pole in $\sigma < 0$ (none in $\sigma > 0$)
LHP RHP
⇒ all poles are on $j\omega$ axis

Some canonical LPR forms:
1st and 2nd Foster
1st and 2nd Cauer

1st Foster = partial fraction expansion of $Z(s)$

2nd " " " " of $y(s)$

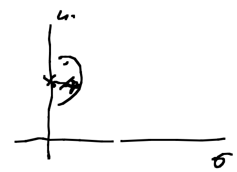
1st Cauer ⇒ remove poles @ ∞ in a continued fraction expansion

2nd " " " poles @ 0
" " "

such a pole has an partial expansion & near a pole @ $s = j\omega$

this is $\underbrace{\dfrac{k}{(s-j\omega_0)^m}} = \dfrac{|k|\, e^{j\angle k}}{|\sigma + j\omega - j\omega_0|\, e^{j\angle \text{pole}}}$

$= r_{\text{atio}}\, e^{j(\angle k - \angle \text{pole})}$



as PR $\quad Re \geq 0 \quad Re = \cos\underbrace{(\angle k - \angle \text{pole})}$

≥ 0 if $-\pi/2 < \angle < \pi/2 \Rightarrow m = 1,\ \angle k = 0$

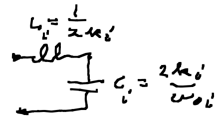
for an LPR driving point $Y(s)$ or $Z(s)$ the poles are simple

& residue k = real

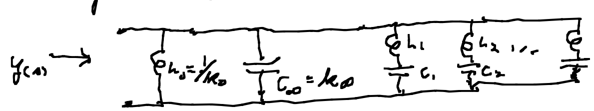
$$Y(s) = \underset{\substack{\uparrow \\ \text{an } L}}{\frac{k_0}{s}} + \underset{\substack{\uparrow \\ \text{a } C}}{k_\infty s} + \sum_i^{\text{finite}} \Big(\underbrace{\frac{k_i}{s - j\omega_{0i}} + \frac{k_i}{s + j\omega_{0i}}} \Big) = \frac{k_0}{s} + k_\infty s + \sum \frac{2 k_i s}{s^2 + \omega_{0i}^2} \Leftarrow \text{odd in } s$$

$$\frac{2 k_i s}{s^2 + \omega_{0i}} = Y_i .$$

$$Z_i = \frac{1}{Y_i} = \frac{s^2 + \omega_{0i}}{2 k_i s} = \frac{s}{2k_i} + \frac{\omega_{0i}}{2 k_i s} =$$

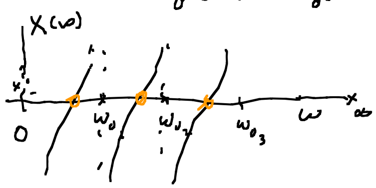


LPR design via $Y(s)$



$j\omega$ axis

$$y(j\omega) = \frac{k_0}{j\omega} + k_\infty j\omega + \sum \frac{2k_i j\omega}{-\omega^2+\omega_{0_i}^2} \Rightarrow \frac{y(j\omega)}{j} = X(\omega) = -\frac{k_0}{\omega} + k_\infty \omega + \sum \frac{2k_i\omega}{-\omega^2+\omega_{0_i}^2}$$



$$0 \le \frac{dX(\omega)}{d\omega} = \frac{k_0}{\omega^2} + k_\infty + \sum \underbrace{\frac{2k_i}{-\omega^2+\omega_{0_i}^2} + \frac{2k_i\omega\cdot 2\omega}{(-\omega^2+\omega_{0_i}^2)^2}}_{\frac{2k_i\omega^2+2k_i\omega_{0_i}^2}{(-\omega^2+\omega_{0_i}^2)^2}}$$

$\Downarrow$

between 2 poles there is a zero.

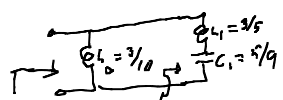
Ex: $$y(s) = \frac{5(s^2+2)}{s \times (s^2+3)} = \frac{k_0}{s} + \frac{k_1 s}{s^2+3}$$

$k_0 \Rightarrow \times s \Rightarrow s\,y(s)\Big|_{s=0} = \frac{5(s^2+2)}{s^2+3}\Big|_{s=0} = k_0 + \underbrace{\frac{k_1 s^2}{s^2+3}\Big|_{s=0}}_{0}$

$\delta(y(s)) = 3$ " degree of y

$$= \frac{10/3}{s} + \frac{(5/3)s}{s^2+3} \quad y_1$$

$\Rightarrow k_0 = 5 \times \frac{2}{3} = 10/3$



$L_1 = 3/5$, $C_0 = 3/10$, $C_1 = 5/9$

$y(s)$ 2nd Foster

$$y_1 = \frac{5s/3}{s^2+3};\ \frac{1}{y_1} = z_1 = \frac{s^2}{5s/3} + \frac{3}{5s/3}$$

$k_1 = \times \frac{s^2+3}{s} = \frac{5(s^2+2)}{s^2} = k_0\frac{s^2+3}{s^2} + k_1$

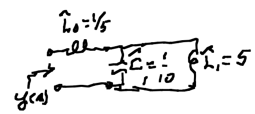
set $s^2 = -3 \Rightarrow k_1 = \frac{5(s^2+2)}{s^2}\Big|_{s^2=-3} = \frac{5(-1)}{-3} = 5/3$

1st Foster $$z(s) = \frac{1}{y(s)} = \frac{s(s^2+3)}{5(s^2+2)} = \hat{k}_\infty s + \frac{\hat{k}_1 s}{s^2+2} = \frac{s}{5} + \frac{\frac{1}{5}s}{s^2+2}$$

$$\hat{k}_1 = \frac{s^2+2}{s}\cdot z(s) = \frac{s^2+3}{5}\Big|_{\substack{s^2+2=0\\ s^2=-2}} = \frac{+1}{5}$$

$\Downarrow$

$$\hat{z}_1 = \frac{\frac{1}{5}s}{s^2+2} \Rightarrow \hat{y}_1 = \frac{1}{\hat{z}_1} = \frac{s^2+2}{1/5\,s} = 5s + \frac{1}{\frac{1}{10}s}$$



$\hat{L}_0 = 1/5$, $\hat{C} = 1$, $\frac{1}{10}$, $\hat{L}_1 = 5$, $y(s)$

1st Foster

1st Cauer: Remove pole @ ∞

$$y(s) = \frac{5(s^2+2)}{s(s^2+3)} \quad \text{no pole @}\infty$$

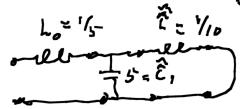
$$z(s) = \frac{1}{y(s)} = \frac{s^3+3s}{5s^2+10} = \frac{s}{5} + \underbrace{\text{remainder } z_2}_{\text{no pole @ } \infty \text{ so turn over}} = \frac{s}{5} + \underbrace{\frac{\frac{1}{5}s}{s^2+2}}_{z_2}$$

$$= \frac{s}{5} + \cfrac{1}{\cfrac{s^2+2}{\frac{s}{5}}}$$

$$= \frac{s}{5} + \frac{1}{5s + 10/s} = \underbrace{\frac{s}{5}}_{z} + \cfrac{1}{\underbrace{5s}_{y} + \cfrac{1}{\underbrace{s/10}_{z} + 0}}$$

continued fraction

1st Cauer



1st Cauer

remove poles @ 0

$y(s)$